

# Constraints on Shift-Symmetric Scalar-Tensor Theories with a Vainshtein Mechanism from Bounds on the Time Variation of $G$

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We show that the current bounds on the time variation of the Newton constant  $G$  can put severe constraints on many interesting scalar-tensor theories which possess a shift symmetry and a nonminimal matter-scalar coupling. This includes, in particular, Galileon-like models with a Vainshtein screening mechanism. We underline that this mechanism, if efficient to hide the effects of the scalar field at short distance and in the static approximation, can in general not alter the cosmological time evolution of the scalar field. This results in a locally measured time variation of  $G$  which is too large when the matter-scalar coupling is of order one.

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Many theories in which gravity is modified with respect to general relativity (GR) contain, in addition to the metric, a scalar field which is coupled directly to matter. Such scalar-tensor theories appear naturally in low energy limits of string theory and are also obtained from phenomenological brane-world constructions (such as the DGP model [1]). Some are also of current interest as able to produce an interesting cosmology via a large distance modification of gravity. In such theories, in contrast to GR, matter not only interacts via the helicity-2 graviton, but also via the exchange of the scalar field. In general, one faces the following dilemma: Either this field is coupled to matter with gravitational strength, as required to produce order one deviations from GR, but then the theory cannot pass local tests of gravity, or the coupling is very small, but then there are no significant effects of the scalar. A canonical example is the Brans-Dicke theory [2, 3] and its extensions [4] whose parameters are tightly constrained by the local tests of gravity and observations of binary pulsars (see for instance [5]).

A way out of this dilemma is provided by the Vainshtein mechanism, first proposed in the context of massive gravity [6, 7] (a proof was recently provided in [8]). Indeed, close to localized bodies, this allows to screen effects which lead to large deviations from GR at large distances. This mechanism was also shown to be present in the DGP brane model [1] as well as its decoupling limit [9]. It was later generalized and shown to apply to a large class of scalar-tensor models, called in [10] “k-mouflage” gravity models, with a non-linear kinetic self-interaction of a scalar field providing a self-screening of the scalar force *à la* Vainshtein (hence the name k-mouflage). This class contains in particular the Galileon model [11], and its covariantized versions [12–15]. Many applications of the Galileon model and its extensions to the late-time acceleration, including minimally coupled [16–19] as well

as non-minimally coupled models [20–22], have been considered, while various constraints coming from cosmology as well as from local observations have been studied [17, 19, 23–25].

In this paper, we point out that in spite of the fact that the Vainshtein screening indeed allows to pass most of the constraints coming from local observations by cutting off the spatial variation of the scalar field near massive bodies, the tests on the constancy of the Newton constant may easily rule out many models. Indeed, we show that in many shift-symmetric models, the evolution with time of the scalar field is (approximately) the same everywhere and it follows its cosmological behavior. If the scalar is directly coupled to matter, this induces a variation of the Newton constant  $G$ , which is tightly constrained by a number of observations (see, e.g., the review [26]). The most stringent bounds come from binary-pulsar data [27] and above all Lunar Laser Ranging experiments [28], the latter giving  $|\dot{G}/G| < 1.3 \times 10^{-12} \text{ yr}^{-1}$ , or in terms of the Hubble value today,  $H_0$ ,  $|\dot{G}/G| < 0.02 H_0$ . As we will see below, the time variation of the scalar field is generically of order of the Hubble scale  $H_0$  (unless it is in the “cosmological” screening regime with a tiny energy scale  $M \ll H_0$ ). This, whenever the direct coupling of the scalar field to matter is of order of one, induces a too large variation of Newton’s constant,  $|\dot{G}/G| \sim H_0$ .

We consider the following general action,

$$S = \frac{M_{\text{P}}^2}{2} \int d^4x \sqrt{-g} (R + \mathcal{L}_s + \mathcal{L}_{\text{NL}}) + S_m [\tilde{g}_{\mu\nu}, \psi_m], \quad (1)$$

where  $R$  is the Ricci scalar of the metric  $g_{\mu\nu}$ ,  $\mathcal{L}_s = -(\partial\varphi)^2$  is the standard kinetic term of a scalar field  $\varphi$  (normalized to be dimensionless),  $\mathcal{L}_{\text{NL}}$  describes some generic nonlinear self-interaction of  $\varphi$ , and the matter fields (collectively denoted as  $\psi_m$ ) are minimally coupled

to the physical metric  $\tilde{g}_{\mu\nu} = \mathcal{A}^2(\varphi)g_{\mu\nu}$ . Because of this coupling, gravity is modified at large distances through a scalar exchange, while GR is supposed to be restored at small distances thanks to the Vainshtein screening effect made possible by the self-interaction  $\mathcal{L}_{\text{NL}}$ . This screening occurs for rather generic nonlinear interaction terms  $\mathcal{L}_{\text{NL}}$  [10] and we do not specify a precise form of it. We however assume that  $\mathcal{L}_{\text{NL}}$  is shift symmetric, i.e., it does not change under the transformation  $\varphi \rightarrow \varphi + \text{const}$ . For example, we can choose  $\mathcal{L}_{\text{NL}}$  to be in the Galileon or k-essence families. For dimensional reasons  $\mathcal{L}_{\text{NL}}$  must contain a mass scale  $M$ . We will assume that this is the only additional scale entering our action. In general, this scale is fixed by phenomenological requirements, e.g. to get present day acceleration of the Universe from Galileons,  $M$  should be of order of the Hubble scale  $H_0$ .

Note that by a conformal transformation, action (1) can always be rewritten in a form where matter is only minimally coupled to one metric  $\tilde{g}_{\mu\nu}$ , going to the so-called Jordan frame. Here we will work rather in the Einstein frame (1), with the understanding that our results would apply to any theory whose action can be put in the form (1) by a suitable field redefinition. We will also not consider effects that can arise, even in theories with minimal coupling to gravity in the Einstein frame, due to the non-linear kinetic coupling of the graviton to the scalar field (called “kinetic braiding” in [16]).

The variation of Eq. (1) with respect to the metric  $g_{\mu\nu}$  gives the (modified) Einstein equations,

$$M_{\text{P}}^2 G_{\mu\nu} = T_{\mu\nu}^{(\text{st})} + T_{\mu\nu}^{(\text{NL})} + T_{\mu\nu}^{(\text{m})}, \quad (2)$$

where  $T_{\mu\nu}^{(\text{st})}$ ,  $T_{\mu\nu}^{(\text{NL})}$  and  $T_{\mu\nu}^{(\text{m})}$  are respectively the energy-momentum tensors for the standard scalar kinetic term, its nonlinear term, and the matter contribution. The equation of motion for the scalar field is

$$\nabla_\mu (\nabla^\mu \varphi + J_{\text{NL}}^\mu) = -\alpha(\varphi) M_{\text{P}}^{-2} T^{(\text{m})}, \quad (3)$$

where  $\alpha(\varphi) \equiv d \ln(\mathcal{A}) / d\varphi$ , the nonlinear current  $J_{\text{NL}}^\mu$  is obtained by variation of  $\mathcal{L}_{\text{NL}}$  with respect to the gradient of the scalar field,  $J_{\text{NL}}^\mu \equiv -\frac{1}{2} \delta \mathcal{L}_{\text{NL}} / \delta \varphi_{,\mu}$ , and  $T^{(\text{m})}$  is the trace of the matter energy-momentum tensor in the Einstein frame. Note that the field equations (3) can always be written as the divergence of a current, because of shift symmetry. We also rescale the Planck mass so that  $\mathcal{A}(\varphi) = 1$  at present. We stress that the trace of the matter energy-momentum tensor defined in the Einstein frame,  $T_{\mu\nu}^{(\text{m})}$ , differs by the factor  $\mathcal{A}^4(\varphi)$  from the trace of the (conserved) Jordan-frame energy momentum tensor. However, as we will see below, the time variation of  $\varphi$  is small (of order of the Hubble scale or less), so that the change of  $\varphi$  with time can be neglected in the r.h.s. of (3), giving only small corrections. For instance, if  $\mathcal{A}(\varphi) = e^\varphi$ , the approximation  $\mathcal{A}^4(\varphi) \approx 1$  is valid for  $|\varphi| \ll 1$ , i.e., for  $|\Delta t| \ll H^{-1}$ .

The cosmological evolution of the scalar field,  $\varphi_{\text{cosm}}(t)$  can be easily read from (3),

$$\ddot{\varphi}_{\text{cosm}} + 3H\dot{\varphi}_{\text{cosm}} - \nabla_0 (J_{\text{NL}}^0) = \alpha(\varphi) M_{\text{P}}^{-2} T^{(\text{m})}. \quad (4)$$

Three different cosmological regimes can be identified. In a regime where the cosmological energy density of the scalar field,  $\rho_\varphi$ , is subdominant compared to the matter energy density,  $\rho_{\text{m}}$ ,  $\rho_\varphi \ll \rho_{\text{m}}$ , the scalar field equation (4) is decoupled from the metric equation (2). Then from the Einstein equations it follows that  $\rho_{\text{m}} = 3M_{\text{P}}^2 H^2$ , thus the r.h.s. of (4) is  $\sim \alpha H^2$ . If the scalar field is away from the “cosmological” Vainshtein regime (i.e., when the nonlinear term in the l.h.s. of (4) is negligible), then from Eq. (4) one can see that a particular solution to (4) is  $|\dot{\varphi}_{\text{cosm}}| \sim \alpha H$ . Notice that the general solution also contains a homogeneous decaying solution  $C_0 \exp(-3 \int dt H)$  with an arbitrary constant  $C_0$ , however unless this constant (or, equivalently, the initial condition) is fine tuned, the time variation of  $\varphi$  remains of order of  $\alpha H$ .

In the second regime matter is again dominant,  $\rho_\varphi \ll \rho_{\text{m}}$ , but the scalar field is in the cosmological Vainshtein regime. Then Eq. (4) contains, in addition to  $H$ , also the “nonlinear” scale  $M$ . Therefore the solution to (4) generically contains a combination of scales  $H$  and  $M$ . When this scale is small with respect to  $H_0$ , the time evolution of  $\varphi_{\text{cosm}}$  may thus be suppressed,  $|\dot{\varphi}_{\text{cosm}}| \ll H_0$ . This is the cosmological analog of the original Vainshtein mechanism.

The third regime is realized in the case when the scalar field is dominant,  $\rho_\varphi \gg \rho_{\text{m}}$ , and in particular when the late-time acceleration of the Universe is driven by the scalar field. In this case, both the metric and the scalar field equations depend only on one dimensionful parameter,  $M$ , which is of order of  $H_0$ . We thus conclude that the typical value of the present variation of the scalar field is the Hubble scale,  $|\dot{\varphi}_{\text{cosm}}| \sim H_0$ .

Let us consider now the local effects caused by the conformal coupling of the scalar field. For a slow time evolution,  $\varphi_{\text{cosm}}(t)$  can be written as the linear approximation,

$$\varphi_{\text{cosm}}(t) = \varphi_{\text{cosm}}(t_0) + \dot{\varphi}_{\text{cosm}}(t_0) t. \quad (5)$$

Note that (5) imposes the boundary value of the field far from localized sources. The solution to the full equation of motion (3) at any point of space-time (including the regions close to massive bodies, in particular, inside the Vainshtein radius) depends on both time and space coordinates. The key observation here is that thanks to the shift symmetry of the equation of motion, the PDE (3) allows separation of variables in the following way,

$$\varphi(t, r) = \varphi(r) + \dot{\varphi}_{\text{cosm}}(t_0) t + \varphi_{\text{cosm}}(t_0), \quad (6)$$

where  $r$  is the distance to the source. It is not difficult to see that the above ansatz (which has also been used in other contexts [29]) “passes through” the full equation of motion (3), giving an ordinary differential equation of the second-order on  $\varphi(r)$ , with possible remnants from the time-dependence in a form of a constant,  $\dot{\varphi}_{\text{cosm}}(t_0)$ . The last two terms in the above ansatz give the boundary condition for the PDE imposed by the cosmological evolution (5), provided we choose the radial dependent part

of (5),  $\varphi(r)$ , to vanish at infinity. Now, the ODE on  $\varphi(r)$  is of the second order, and supplied with two boundary conditions,  $\varphi(r = \infty) = 0$  and  $\varphi'(r = 0) = 0$  (the last one comes from the regularity at the origin), which is in general sufficient to find a unique solution. Provided that this solution is non singular (which is in some cases a strong mathematical requirement), and assuming that (5) is a good approximation for the time-dependent cosmological evolution of  $\varphi$ , our ansatz gives a solution for all times to the field equation (3) for a spherical source centered at  $r = 0$ . The key point of this paper is that the time derivative of  $\varphi(t, r)$  is found to be independent of  $r$ , i.e., it is set by the cosmological evolution even inside the regions where the Vainshtein screening operates.

It is worth mentioning that our ansatz (6) and boundary conditions are in fact selecting a particular class of solutions. Indeed, if formulated in terms of a Cauchy problem, they implies  $\varphi(t = 0, r) = \varphi(r)$  and  $\dot{\varphi}(t = 0, r) = \dot{\varphi}_{\text{cosm}}(t_0)$ . In principle, there is no reason not to choose some radially dependent initial velocity,  $\dot{\varphi}(r) = \mathcal{C}(r)$ . In contrast to (6), such solutions, however, are not stationary, and they should relax to the stationary one, assuming that the latter is stable. A numerical check of this relaxation, as required by the nature of the field equations, goes however far beyond the scope of this paper.

Let us consider a couple of illustrative examples. First, as the non-linear term, we take one of the Galileon Lagrangians, namely,

$$\mathcal{L}_{\text{NL}} = -M^{-2} \square \varphi (\partial \varphi)^2, \quad (7)$$

and the coupling to matter  $\mathcal{A}(\varphi) = e^\varphi$ . It is not difficult to find that the evolution of  $\varphi_{\text{cosm}}$  in two different cosmological regimes is in accord to our general findings (we assume here a vanishing cosmological constant): (i) when  $\rho_\varphi \ll \rho_m$  and the nonlinear term is subdominant in (4) then  $\dot{\varphi}_{\text{cosm}} = -2H$ ; (ii) when  $\rho_\varphi \gg \rho_m$  and the nonlinear term is dominant in (4) then  $\dot{\varphi}_{\text{cosm}}^2 = 2M^2/3$ . The time dependent solution for the scalar field around a body of mass  $m$  is then given by (6) with,

$$\varphi'(r) = \frac{rM^2}{4} \left[ -1 + \sqrt{1 + \frac{16Gm}{M^2 r^3} \left( 1 + \frac{\dot{\varphi}_{\text{cosm}}^2}{2M^2} \right)} \right]. \quad (8)$$

Note that the last piece inside the parentheses is due to the cosmological time evolution of  $\varphi$ . As a second example we consider a scalar field Lagrangian with the the signs of the scalar field kinetic terms flipped with respect to previous example, and the coupling to matter  $\mathcal{A}(\varphi) = e^{-\varphi}$ . This Lagrangian allows self-accelerating solution [16], with  $H^2 = M^2/(3\sqrt{6})$  and  $\dot{\varphi} = \sqrt{6}H$ , while the time-dependent stationary solution for the scalar field is still given by (6) with the same radial-dependent part (8).

The time-dependence of  $\varphi$  leads to a variation of the effective Newton constant with time. This can be seen by making the conformal transformation to the Jordan (physical) frame, with the metric  $\tilde{g}_{\mu\nu}$ . Generically, there

are two effects, which enter the final result for the evolution of Newton's constant: the exchange of helicity-0 modes and the rescaling of the coordinates via the conformal transformation of the metric. It should be noted that in standard scalar-tensor theories (without screening mechanisms), these effects are of the same order, so that they even can compensate each others (as in Barker's theory [30]) giving no change of  $G$ . In our case, however, the exchange of  $\varphi$  is screened by the Vainshtein mechanism, so that only one effect — the stretching of coordinates — is important. As a result, the effective Planck mass in the action gains a dependence on  $\varphi_{\text{cosm}}(t)$ ,  $\tilde{M}_{\text{P}} = \mathcal{A}^{-1}(\varphi_{\text{cosm}}) M_{\text{P}}$ . Thus, the observed Newton constant evolves with time as

$$|\dot{G}/G| \approx 2\alpha \dot{\varphi}_{\text{cosm}}(t). \quad (9)$$

As we have seen before, depending on the regime,  $|\dot{\varphi}_{\text{cosm}}| \sim \alpha H$  (in the matter domination regime) or  $|\dot{\varphi}_{\text{cosm}}| \sim H$  (when the scalar field is dominant). Therefore, presently one has  $|\dot{G}/G| \sim \alpha^2 H_0$  if the scalar field is subdominant and away from the cosmological screening, and  $|\dot{G}/G| \sim \alpha H_0$  when the scalar field is dominant, in particular, when it drives the late-time acceleration of the universe. This applies in particular for a constant  $\alpha$ , i.e., a conformal coupling  $\tilde{g}_{\mu\nu} = e^{2\alpha\varphi} g_{\mu\nu}$ .

The observational constraints from Lunar Laser Ranging give  $|\dot{G}/G| < 0.02H_0$  which is enough to rule out theories of the kind considered here with a scalar coupling to matter of the order of the gravitational one (i.e.,  $\alpha \approx 1$ ). In order for a theory to explain the accelerated expansion of the universe at present days, and pass the constraints on the variation of  $G$ , one should assume  $\alpha < 0.01$ . It is interesting to note that a similar constraint on the matter-scalar coupling constant was obtained for the covariant Galileon theory from a combined analysis of supernovae, baryonic acoustic oscillations and cosmic microwave background [22].

Let us now briefly discuss the case of non shift-symmetric theories. There is a class of such theories which can be put in the form (1) by suitable field redefinitions, in which case our conclusions apply. When this is not the case, the situation must be carefully re-analyzed. Indeed, the fact that the ansatz (6) leads to a mere ODE to solve for  $\varphi(r)$  is a direct consequence of the shift symmetry and it might be that some screening of the time variation of  $G$  occurs when this symmetry is lost. For example, we may introduce a mass term in the action,  $m^2 \varphi^2$ , which explicitly breaks the shift symmetry. If the mass is big enough (say, much bigger than the present Hubble scale,  $H_0$ ), then the cosmological evolution of  $\varphi$  is suppressed, because the scalar follows the minimum of the effective potential,  $\dot{\varphi}_{\text{min}} \sim \alpha H \dot{H}/m^2$ . However, such theories do not possess either an interesting self-accelerating scenario driven by the kinetic term.

We also stress that even in the shift-symmetric case, it might be that the ODE obeyed by  $\varphi(r)$  does not possess regular solutions, or that it leads to a stationary solution (6) which happens to be unstable as a solution of the

PDE (3). If so, it may open a way out of our conclusions, necessitating the use of a more general ansatz than (6) to solve the field equations, which could in turn result in a Vainshtein suppression of the time variation of  $G$  in the solar system.

It is also interesting to mention another possibility of avoiding any significant time evolution of  $G$ , namely, to violate our assumption of a conformal matter-scalar coupling. In particular, in the relativistic MOND theory, called TeVeS [31], where the physical metric is related to the Einstein one in a disformal way, the time variation of the Newton constant is strictly zero [32]. This is a consequence of the different scalings of time and space coordinates with  $\mathcal{A}(\varphi_{\text{cosm}})$  when one imposes that the physical metric  $\tilde{g}_{\mu\nu}$  tends to the Minkowski metric at spatial infinity. This absence of time variation of  $G$  also applies to other theories with disformal coupling, in particular to the improved relativistic MOND [33], where the k-mouflage screening has been used to pass solar-system and binary-pulsar constraints.

Let us finally underline that the DGP brane model, although equivalent to a scalar-tensor theory of the Galileon type in the UV (in particular in the decoupling limit [9]), is not fully described by such a theory at cosmological scales (IR limit). Therefore our analysis does not apply to the DGP model, and this explains why Ref. [34] did not find any local time variation of  $G$ .

To conclude, we have shown that a generic scalar-tensor theory with conformal coupling of a scalar field to matter, and with a shift symmetry of the scalar La-

grangian, is tightly constrained by the bounds on time variation of the Newton constant. The models which fall into this category are not only standard (massless) Brans-Dicke-like theories, but also those featuring a Vainshtein screening mechanism due to the kinetic self-coupling, in particular non-minimally coupled Galileon models. We argued that the local time evolution of the scalar field is set by its cosmological evolution and not screened by the Vainshtein effect, which is only able to suppress the “fifth force” due to the exchange of helicity-0 degree of freedom. The time derivative of the scalar field is of order of the Hubble scale, unless the whole universe is in the screening regime (“cosmological” Vainshtein effect with  $M \ll H_0$ ). This induces a time evolution of Newton’s constant of the same order. This result applies for both the matter domination and the scalar field domination cosmological regimes. It also does not depend on a particular form of the non-linear self-interaction term, provided it is shift-symmetric. The key point is that the time dependence in (6), which is crucial in (9), is the same irrespectively of the precise structure of  $\mathcal{L}_{\text{NL}}$ . The evolution of the Newton constant is, however, tightly constrained by observations,  $|\dot{G}/G| \lesssim 0.02H_0$ , therefore the conformal coupling on such theories is constrained too. For example, if the non-minimal coupling is of the form  $\exp(2\alpha\varphi)$ , then  $|\alpha|$  should be less than  $10^{-2}$ .

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